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# Transformation of trigonometric functions integral calculus examples

This section: 4. Full trigonometric functions 4. Integral trigonometric functions Recalling the definition of an anti-derivative that, if $-fracs f(x) f(x) -g(x),$ then $-int g(x) dx-f(x) - C.$ C.$that is, every time we have a differentiation formula, we get an integration formula for nothing. Here is a list of some of them. Derived Rule Anti-Derivative Rule $'frac'dx 'sin x' 'cos x$'int 'cos x' dx 'sin x ' C$'frac'd'dx 'cos x' -sin x $$'int 'sin x' dx ' 'cos x' 'C$'frac'dx 'tan x 'sec'2x$'s $'s 'dry'2x' dx 'tan x' 'C$'frac'd'dx cotan x - 'cosec' 2 x $ $'int 'cosec'2x' dx --cotan x 'C$$'frac'd'dx 'sec x 'sec x 'tan x $'int ('sec x'tan x ' dx 'sec x ' C$'$'frac'dx 'cosec' x 'cosec x 'cotan x $'int ('cosec x' dx -'cosec x' -'cosec x' we have developed formulas for anti-derivatives of $sin x$ and $-cos x.$ Question What about the other four? Answer We will get some of them below, and leave others to the exercise set. (Some of them have already appeared as derivatives in Exercise Set 3...). Example 1 Calculate the following. (a) $-int (3-sin x - 4-sec-2x) dx$ (b) $-int 'cos(2x - 6) dx$ (c) $-int 'sin x'cos'2x' dx$ (d) $'int 'tan x' dx$ Solution (a) Consultation of the table above, $'int (3-sin x - 4'sec-2x) dx $$'$'s $'3'sin x' dx - 'int 4'sec-2x' dx $ $3 'int's 'sin x' dx - 4 'int 'dry' dx$ (full-life properties) $-3-cos x - 4-tan x -C$ (from the table) (b) The calculation of $$2x -6)dx requires substitution: $$14-blue-u -2x-6$$$-color We now have{1}{2} $'10t 'cos(2x) $$$$'s $$'s 'cos' and 'frac{1}{2}{1}{2} 'using substitution' $$$1{1}{2} $$$$$$$$$$$$1{1}{2} 'sin' (from the table) $$'s 'frac{1}{2} 'sin(2x-6) - C.$ (using substitution) (c) This one can also be done using a substitution. The trick is to replace the term $'cos x$ as follows: $'blue's 'cos x$' '$' 'blue'of 'dx' -sin x $$$$$-blue-dx - 'frac{1}'sin x' from $$ $$$$$$$20 'int'2 'left'{1} 'sin x'right) $ $- $- $2 of $- 'frac'3 -{3} 'C$$' - frac-cos-{3} C.$ (using substitution) (d) Write $ $$19-tan x-dx$as $$00(int) dx,$ and use the same substitution as in part (c): $'int's 'tan' dx$$ frac-sin x'cos x' dx$' 'int 'frac'sin x'u', 'left{1}'sin x 'right) of the $ $- 'int 'frac{1} of the $$ - 'ln' C$ $ - 'ln' 'cos x' C.$ (using substitution) Before continuing ... The method in part (b) gives us the following formulas more As a general rule of full rule $'int 'cos x' dx 'sin x' C$ $'int 'cos(ax-b) dx 'frac{1}'a' 'sin(a)x-b) - C$ $ 'sin x' dx - C$$-int 'sin (ax-b) dx - 'frac{1}'a' 'cos(ax-b) - C$ Don't keep keep here are the anti-decirivatives of the six trigonometric functions. (You'll get them in the exercises.) As a general rule of full rule $'int 'cos x' dx 'sin x' C$ $'int 'cos(ax-b) dx 'frac{1} 'a' 'sin(ax'b) - C$$'int 'sin x' dx -cos x - C$$-int 'sin (ax-b) dx - 'frac{1}'a' 'cos(ax-b) - C$ $'int 'tan x' dx - 'ln' C$$-int 'tan (ax-b) dx -frac{1} 'a' 'ln' 'cos (ax-b) C$ $$int 'cotan x' dx 'ln's 'i'n' x '$'int $'int 'cotan(ax-b) dx 'frac{1} 'a' 'ln'sin (ax-b) C$$$'int 'dry x' dx 'ln's 'dry x ' 'tan x' C$$-int -dry (ax-b) dx -frac{1} 'a' 'sec(ax-b) 'tan (ax-b) C$ $'int 'cosec x' dx -'ln'cosec x 'cotan x' C$$-int -cosec (ax-b) dx - 'frac{1}'a' 'cosec(ax-b) - 'cotan(ax-b)' Example 2 Total sales of Ocean King Boogie boards are given $by s(t) - 1-500 -0/6) - $2,000, where $$t is time-to-month, and $t -$0 is January 1. Estimate total sales in the four-month period beginning March 1. Solution Since total sales are given by the full defined monthly sales for the given period ($t -$2 to $$t - $6), we have, in the table above, Total Sales $$$$-int_{3}-{6} [1,500-sin (t-7)/6) - 2,000] dt$$$$$$$$$1,500 ($$${6} $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$ (-pi)/6) - $2$$$$$$$$$ We can also use the tabular part integration method discussed in section 7.1 of Real-World Computing, or section 14.1 of mathematics and finite calculations applied to the real world. Example 3 Assess the following integrals (a) $$$2-2x-1)-sin (x/2) dx$ (b) $2x-cos (3x) dx$ Solution (a) Since repeated differentiation of the first term $(3x-2x-1)$ results in zero, we place it in the D column: D I $$$$3x-2x-1$$$$$(x/2)$ $-$$6x-2$$-2$2-cos (x/) 2)$$$$$$6$$-4-sin (x/2)$$$$0 $8$8-cos (x/2)$ That gives: $$3-2-2x-1)-sin (x/2) dx -2 (3x-2-2x-1) -cos (x/2) - 4 (6x-2) -sin (x/2) - 48-cos (x/2) - C$ (b) Repeated differentiation does not destroy either term. In fact, no matter what term we place in the D column, so let's place the trigonometric function there: D I $$$$$$$$$$$$$$3x$e $-$$$$$3$$$$$$$$9-cos (3x)$ Dx - $$$ (O'1) frac{1}{2}$ $e$$$$$$(3}{4}$$e-2x-sin (3x) -$$$$3{9}{4}$$$$$int e-2x-cos (3x) dx$ (We'll add the integration constant once we're done.) Note that we ended up with the same integral on the right as the one we started with. Calling this integral, I give: $-color-red-I-$$${1}{2}$$e-2x -3x -$$$frac{3}{4}$$e-2x-sin (3x) - and we can now solve for I: $I $$${9}{4} $I $$${1}{2} $e $2x-cos (3x) -$$$${3}{4}$$e-2x-sin (3x),$i., $'frac{13}{4}$$$I '$$'frac{1}{2}$$e$2x-cos (3x) '$$$'frac{3}{4}$$e'2x-sin (3x),$ so that $interest dx) dx -I $$${4}{13}$$$[$$$$${1}{2}$$e-2x-cos (3x) -$$$frac{3}{4}$$e-2x-sin (3x)$$$$$$$$$$$$$$ suggestions for improving this resource. Email us to: Stefan Waner (matszw@hofstra.edu) Steven R. Costenoble (matsrc@hofstra.edu) Last update: March 1997 Copyright © 1997 StefanWaner and Steven R. Costenoble Mobile Notice You appear to be on a device with a narrow screen width (i.e. you're probably on a mobile phone). Due to the nature of mathematics on this site, it is preferable to views in landscape mode. If your device is not in landscape mode, many equations will run on the side of your device (should be able to scroll to see them) and some menu items will be cut off due to the narrow width of the screen. Review each of the following integrals. View Mobile Review Show All Notes Hide All Notes Mobile Reviews You appear to be on a device with a narrow screen width (i.e. you're probably on a mobile phone). Due to the nature of mathematics on this site, it is preferable to views in landscape mode. If your device is not in landscape mode, many equations will run on the side of your device (should be able to scroll to see them) and some menu items will be cut off due to the narrow width of the screen. In this section, we will look at a number of integrals involving trig functions and some of the techniques we can use to help us evaluate them. Let's start with an integral that we should already be able to do. ['begin'aligning' x-sin '5 x',dx' and 'int'5',du'hspace'0 .25in'm This{1}{6} integral is easy to do with a substitution because the presence of cosine, however, is easy to do with a substitution. what about the next integral. Example 1 Assess the next integral. Show The solution This integral no longer has the cosine in it that would allow us to use the substitution we used above. Therefore, this substitution will not work and we will have to find another way to do it in its entirety. Remarquons d'abord que nous pourrions écrire l'intégrale comme suit, \[\int{{{{{\sin }^5}x\,dx}} = \int{{{{{{{sin }^4}x\,\sin x\,dx}} = \int{{{{{left( {{{\sin }^2}x} \right)}^2}\,\sin x\,dx}}}] Maintenant, rappelez-vous l'identité trig, \[{\cos ^2}x + {\sin ^2}x = 1\hspace{0.25in}\,\, \Rightarrow \hspace{0.25in}\,\,{\sin ^2}x = 1 - {\cos ^2}x\] Avec cette identité, l'intégrale peut être écrite comme, \[\int{{{{{sin }^5}x\\ ,dx}} = \int{{{{\left( {1 - {{\cos }^2}x} \right)}^2}\,\sin x\,dx}}}] et nous pouvons maintenant utiliser la substitution \(u = \cos x\). That gives us, [[begin'aligning') -5-x-1,dx -int 'left' '(1 - '2' 'right')-from '1' - '2' 'left' (u - 'frac{2}{3}'3' -frac{1}{5} '5'5'right) - 'cos x' - 'frac{2}{3}'3'x - 'frac{1}{5}'cos '5'x 'c'end'align' with a bit of rewriting on the integrand we were able to reduce this to a fairly simple substitution. Note that we were able to do the rewrite that we did in the previous example because the exhibitor on the sinus was In these cases, all we need to do is strip one of the sines. The exhibitor on the remaining sines will then be equal and we can easily convert the remaining sines into cosines using the identity, [begin-equation '2'x', 'sin', '1 label'eq:eq1' if the exhibitor on the sines had been difficult to do. We could strip a sinus, but the remaining sines would then have an odd exhibitor and while we could convert them into cosines the resulting integral would often be even more difficult than the original integral in most cases. Let's take another example. Example 2 Assess the next integral. So in this case we have both sines and cosines in the problem and in this case the exhibitor on the sinus is even while the exhibitor on the cosineer is strange. Thus, we can use a similar technique in this integral. This time, we're going to strip a cosine and convert the rest into sinus. ['begin'align'6 x 'cos'3 x,dx' Int's 6 x 'cos'2 x,'cos x,dx It's not going to be the last time it's not going to be the first time it's a Sin '6' x'left( '1 - 'sin '2'x 'right',cos x,dx'hspace', '0.5in'u 'sin x' 'int' '6'left( 'right',from 'int' - '8',', from 'frac{1}{7} 'sin', '7'x - 'frac{1}{9}'sin '9'x 'c'end'aligned , let us pause for a second to summarize what we have learned so far about the integration of sinus and cosine powers. In this integral if the exhibitor on the sines is strange, we can strip a sinus, convert the rest into cosins using 'eqref-eq:eq1') and then use the substitution .A. Similarly, if the exhibitor on the cosines is strange, we can strip a cosine and convert the rest into sines and use the substitution (u -sin x). Of course, if both exhibitors are strange, then we can use either method. However, in these cases, it is usually easier to convert the term with the smaller exhibitor. The only case we haven't looked at is what happens if the two exhibitors are even? In this case, the technique we used in the first two examples just won't work and in fact there really isn't a single defined method for doing these integrals. Each integral is different and, in some cases, there will be more than one way to be an integral part. That being said, most, if not most, integrals involving sine and cosin products in which both exhibitors are even can be made using one or more formulas to rewrite the integrand. [begin'aligns '2' x 'frac{1}{2} 'left' '(1' 'cos'left( '2x' 'right') 'sin ' '2' x 'and 'right')'{1}{2} The first two formula{1}{2}s are the standard half-angle formula of a trig class written in a form that will be more practical for us. The last is the standard two-angle formula sinus, again with a little rewrite. Let's take an example. Example 3 Assess the next integral. View the solution As shown above there is often more than one way to be an integral part in which both exhibitors are even. This integral is an example. There are at least two solution techniques for this problem. We will do both solutions starting with what is probably the longest of the two, but it is also the one that many people see first. Solution 1In this solution, we will use the two half-angle formulas above and simply replace them in the integral. [[Begin-aligning '2 x 'cos', '2 x', dx', 'int'frac{1}{2} left( '1 - 'cos 'left( '2x' 'right)left ('frac{1}{2}' right) left ('1' So we still have a completely-made integral{1}{4}, so we still have a completely-made, However, , note that we have managed to reduce the integral to a single term causing problems (a cosine with a power) rather than two terms causing problems. In fact, to eliminate the rest of the term problem all we have to do is reuse the first half-angle formula given above. [[Begin'aligning' '2 x 'cos', '2 x', ' frac{1}{4} 'int'1 - 'frac{1}{2}'left ('11' 'cos'left('4x'right)',dx'- 'frac'{1}{4}{1}{2} {1}{2} - 'frac{1}{2}'co 'left', dx 'frac{1}{4}'left(frac{1}{2}x - 'frac{1}{2}x' - 'frac{1}{8}'sin 'left(4x' right) So{1}{8}s{1}{8} {32}s-4x -4x-right- 'fra{1}c{1}{8}x' , this solution required a total of three trig identities to complete. Solution 2In this solution, we will use the half-angle formula to help simplify the entire system as follows. ['begin'aligning' '2 x 'cos',dx' 'left' ('sin x'cos x 'right')2,dx 'int'left {1}{2}'s 'left'(2x'-right)','2 ',dx', 'frac{1}{4}'int's '2'left ('2x' 'right)', dx'end-align' Now, we use the dual-angle sinus formula to reduce to a integral that we can do. ,dx{1}{8} '1' - 'cos 'left( '4x' 'right',dx' -frac{1}{8}x - 'frac{1}'{32} 'sin left' ('4x' 'right) - lining end] This method required only two trigs to complete the Note that the difference between these two methods is more that of disorder. The second method is not significantly easier (other than needing one less trig identity) it is simply not as messy and will often result in an easier process. In the previous example, we saw two different methods of solution that gave the same answer. that this will not always happen. In fact, more often than not, we will have different answers. However, as discussed in the Part Integration section, the two responses will differ only one time. In general, when we have sine and cosin products in which both two are even we will need to use a series of half-angle and/or double angle formulas to reduce the integral in a form that we can incorporate. In addition, the larger the exhibitors, the more we will need to use these formulas and therefore the messier the problem. Sometimes in the process of reducing integrals in which the two exhibitors are, even we will run through the sinus and cosine products in which the arguments are different. These will require one of the following formulas to reduce the products to the integrals that we can make. ['begin'lining'' 'alpha 'cos'and 'beta{1}{2}'s 'left' ('alpha' - 'beta') 'sin'left('alpha', 'beta') 'right') 'sin' 'frac{1}{2}'left' - 'cos'left( 'alpha' - 'beta' 'right') - 'cos'left('alpha', 'beta', 'right') Frac{1}{2} left - 'alpha' - 'beta', 'right') 'cos'left('o'alpha', 'beta', 'right'] Let's take a look at an example of one of these types of integrals. Example 4 Assess the next integral. Show the solution This integral requires the last formula listed above. \[\begin{align*}\int{{\cos \left( {15x} \right)\cos \left( {4x} \right)\,dx}} & = \frac{1}{2}\int{{\cos \left( {11x} \right) + \cos \left( {19x} \right)\,dx}}\\ \ \ \ \\\\